The travelling salesman problem on a dilute lattice: a simulated annealing study

COMMENT

The travelling salesman problem on a dilute lattice: a simulated annealing study

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Abstract. We have studied the travelling salesman problem on a dilute square lattice of size 15×15, using the simulated annealing technique. We determined the average optimised travel distances per city α_E and α_C as functions of lattice site occupation (city) concentration p, using Euclidean and 'Cartesian' type metrics (denoted by E and C respectively) for the calculation of travel distances. $\alpha\sqrt{p}$ is found to have a monotonic variation from a constant A to 1 as p varies from 0 to 1 with $A_E = 0.80 \pm 0.05$ and $A_C = 1.00 \pm 0.05$. We also show that the ratio α_C/α_E reduces from about 1.27 for $p \rightarrow 0$ to 1 for $p = 1$, indicating $A_C \simeq (4/\pi)A_E$.

Following the recent formulation of the travelling salesman problem (TSP) on randomly diluted lattices (Chakrabarti 1986, Dhar *et al* 1987), we study here the variation of the average optimised travel distance per city against the occupation concentration of the cities, using the simulated annealing technique (Kirkpatrick *et al* 1983, Kirkpatrick 1984). In TSP on dilute lattices, cities are represented by the occupied lattice sites and they are randomly distributed with concentration p. The main object is to find the optimised (shortest) route of travel of a salesman who must visit each of the cities at least once and comes back to his starting point. The specific quantities of interest are the average optimised travel distance $\alpha(p)$ per city and also the average degeneracy (entropy) for such optimised tour length. We study here the variation of $\alpha(p)$ against p for a 15×15 square lattice, using the simulated annealing technique.

Dhar *et al* (1987) studied various inequalities for $\alpha(p)$ and suggested that $\alpha(p)\sqrt{p}$ will have a monotonic variation from 1 (lattice constant normalised to unity) for $p = 1$ to a constant A (the value of which depends on the metric used) for $p \rightarrow 0$. Following Armour and Wheeler (1983), one can find an upper bound for the optimal path length when the country (here the lattice) is divided into strips of arbitrary width and the salesman traces a 'directed path' (Chakrabarti 1986) in visiting the cities within each strip and saves only near the edges of the strips. The total saving then can be maximised with respect to the strip width (single parameter variation), giving an estimate of A. For Euclidean metric ($r = x^2 + y^2$) on a lattice Armour and Wheeler (1983) gave $A_E \leq 0.921$. When the travel distance is measured along the lattice edges ($r = |x| + |y|$), Dhar *et al* (1987) gave $A_C \leq \sqrt{4/3}$ for the corresponding value of A in such a 'cartesian' type metric. The simulated annealing technique gives sub-optimal routes near the optimal one for each lattice configuration (city distribution) at each p. After configurational averaging over α we get $\alpha_E(p)$ as well as $\alpha_C(p)$ for both kinds of metric, and we get $A_E = 0.80 \pm 0.05$ and $A_C = 1.00 \pm 0.05$. We also show how the ratio $\alpha_C(p)/\alpha_E(p)$ reduces from around $4/\pi = 1.27$ for $p \rightarrow 0$ (because of random orientation of the
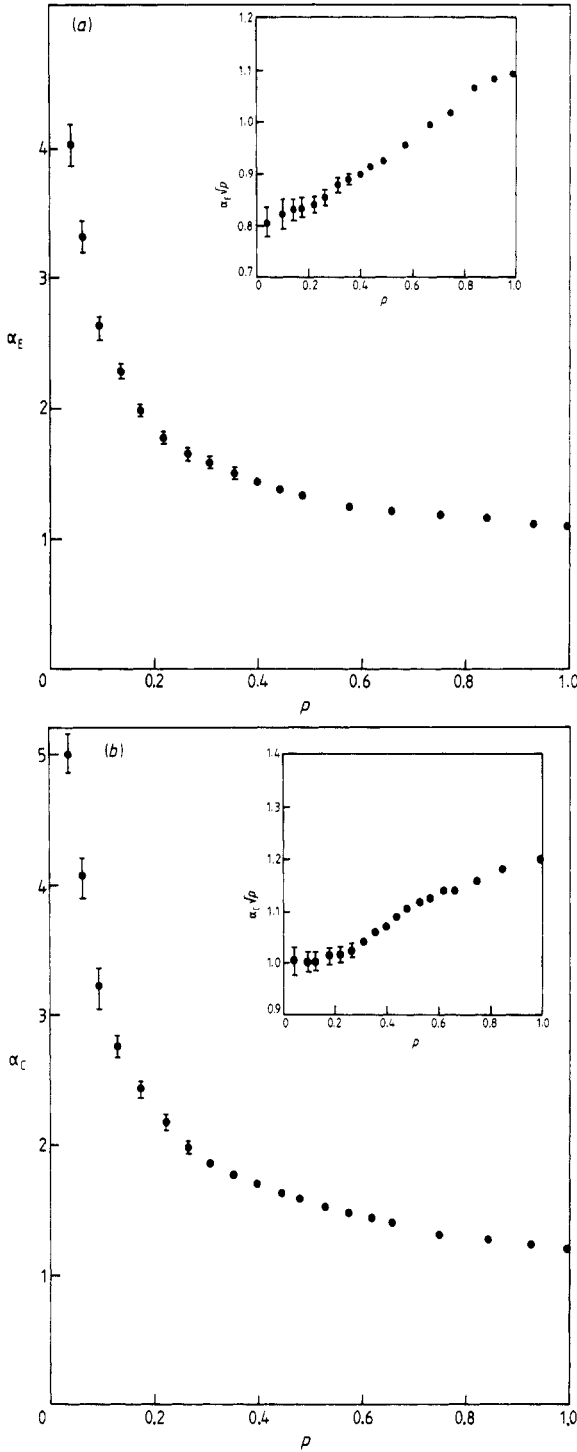
Figure 1. Plot of the simulated annealing results for the average optimised travel distance per city $\alpha(p)$ against the occupation concentration of lattice sites (city concentration) p on a 15×15 square lattice. The insets show the plot of $\alpha\sqrt{p}$ against p, (a) using Euclidean (E) metric and (b) using cartesian (C) metric.

Euclidean TS path with the lattice axes for the cartesian TS path (Dhar *et al* 1987) to 1 for $p \to 1$ (when both paths coincide).

We generate the randomly dilute lattice (15×15 square) configuration following the standard Monte Carlo procedure. For each such lattice configuration, the sub-optimal tour configuration is obtained using the standard simulated annealing procedure (Kirkpatrick 1984) bringing equilibrium at a fairly high temperature (above the 'melting' point) and then reducing the temperature following a constant ratio ($=0.9$ here). In order to avoid the complications of 'dead slow' annealing near the 'glass transition' point, which changes with p and also depends very much on the lattice configuration, we have chosen to maintain the same ratio for temperature reduction throughout. For each lattice configuration 10 to 20 sub-optimal paths at $T \to 0$, starting from randomly different initial tour configurations were obtained and we took the minimum of them (noting their average value also) (see also Rees and Ball 1987). This procedure gives a slight overestimate in the value of α for the travel distance per city for each lattice configuration. This may be noted from figures 1(a) and (b) for $\alpha(p)$ at $p = 1$, where the lattice is perfect and $\alpha(p)$ should strictly be unity, although the above kind of constant ratio annealing schedule gives about 10 to 20% excess (in α_E and α_C respectively) over unity, which can be obtained using proper annealing near the glass transition point. In fact, this excess value in α is a maximum for the ordered distribution of cities near $p = 1$. (This seems to be suggested by experience; indeed $\alpha \sqrt{p}$, which is the real optimised quantity, is itself theoretically expected to be smaller than unity for disordered lattices compared to its unit value for ordered lattices.) Averaging over 10 to 15 lattice configurations at each p, $\alpha(p)$ were determined for both kinds of metric. Figures 1(a) and (b) show the plot of $\alpha_E(p)$ and $\alpha_C(p)$ respectively against p and the insets show the corresponding values of $\alpha \sqrt{p}$ (the error bars indicate the configurational variations). The variation of $\alpha \sqrt{p}$ is observed to be monotonic and for $p \to 0$ they indicate $A_E = 0.80 \pm 0.05$ and $A_C = 1.00 \pm 0.05$. It may be noted that this dilute limit corresponds to continuum and the results of Bonomi and Lutton (1984) and Randelman and Grest (1986) for simulated annealing studies on continuum indicate $A_E \simeq 0.75$. In figure 2, we have plotted the ratio α_C / α_E against p, which shows how the ratio decreases from around 1.27 ($\simeq 4/\pi$) for $p \to 0$ (indicating random orientation of the Euclidean TS path compared to the cartesian TS path in this limit) to about 1.1 (which should strictly be unity, when both TS paths coincide) for $p \to 1$ and this suggests $A_C \simeq (4/\pi)A_E$ (Dhar *et al* 1987).
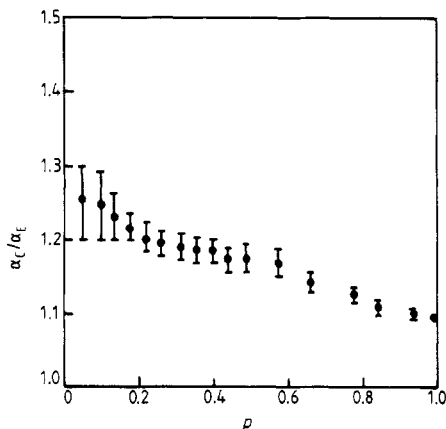

Figure 2. Plot of $\alpha_C(p)/\alpha_E(p)$ against p.

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